

## Transformer Ratio Enhancement using a Ramped Bunch Train in a Collinear Dielectric Wakefield Accelerator

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### ABSTRACT

We propose an experiment to achieve a transformer ratio greater than 2 in a collinear dielectric wakefield accelerating structure. It is known that the transformer ratio cannot generally be greater than 2 for symmetric bunches in a collinear wakefield device. In this experiment, a train of 'drive' bunches, with a particular spacing and charge distribution, excite a structure wakefield used to accelerate a trailing 'witness' beam. By measuring the energy of the drive bunch train and mapping out its wakefield we can demonstrate a transformer ratio of approximately 8 after 4 drive bunches.

### I. INTRODUCTION

In general, the wakefield theorem [1] restricts the maximum accelerating field behind the drive bunch in a wakefield accelerator to be less than twice the maximum retarding field inside the drive bunch thus limiting the efficiency which can be obtained. One of the key concepts central to the physics of wakefield acceleration is the transformer ratio,  $R$ , defined as  $R = (\text{Maximum energy gain behind the bunch})/(\text{Maximum energy loss inside the drive bunch})$ . For the case of a collinear drive and witness beam geometry device,  $R$  is less than 2 except in a few special cases.

Several schemes have been purposed to obtain  $R > 2$  in collinear wakefield structures, but no experimental results have demonstrated this do to the difficulties inherent in these schemes. One of the more promising schemes

involves creating an asymmetric axial current distribution [2] of an individual drive bunch while another interesting scheme tailors the profile of the entire train of drive bunches [3] to produce  $R > 2$ . The fundamental condition for both of these schemes is that all driving particles experience the same decelerating field, whether the particles are in same bunch or in different bunches.

The difficulty with schemes that purpose to use an asymmetric axial drive beam distribution to achieve  $R > 2$ , arises from the lack of suitable techniques to tailor the axial distribution of the drive beam. Although attempts have been made to tailor the longitudinal profile of electron bunches from photoinjectors, this has proved a difficult task.

It is the later technique that we will consider in this paper. For a proof of principle

experiment, we purpose to send a train of 4 electron bunches, with charge increasing from one bunch to the next, through a dielectric lined waveguide. By measuring the energy loss of the drive beam and using a witness beam to measure the peak field behind the bunch we will demonstrate  $R > 2$ .

## II. ENHANCED TRANSFORMER RATIO THEORY

Reference [3] has devised a scheme for enhancing  $R$  beyond 2 in a collinear wakefield accelerator. The scheme works by simple linear superposition of the fields from a train of drive bunches using a clever arrangement of drive bunch spacing and charge. A train of  $N$  drive bunches (Fig. 1), spaced length  $d = (m + \frac{1}{2}) * \lambda_0$  apart can have a transformer ratio after the  $n^{\text{th}}$  bunch,

$$R_n = (n+1)*R_0 \quad (n = 1, 2, \dots) \quad (1)$$

where  $R_0$  is the transformer ratio after the first bunch,  $m$  is an integer multiple of the fundamental and  $\lambda_0$  is the fundamental wavelength of the structure.

Equation (1) represents the maximal growth rate of the transformer ratio and is met only under certain conditions. The most basic of these conditions aside from the spacing is that the magnitude of the charge increases according to,

$$Q_n = Q_0 * [R_0 * n + 1] \quad (n=1, 2, \dots) \quad (2)$$

where  $Q_0$  is the charge in the first drive bunch. Therefore, if  $Q_0 = 1$  nC and  $R_0 = 2$ , then  $Q_1 = 3$  nC,  $Q_2 = 5$  nC, etc. A typical drive bunch distribution is summarized in Fig. 1.

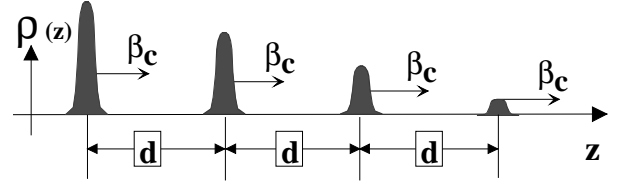


Figure 1) Ramped drive bunch train used to excite wakefields in the dielectric structure.

If the train is constructed according to (2), the maximum  $R$  that can be obtained after the  $N^{\text{th}}$  bunch is  $R_N = R_0 * N$ ; according to (1). This means that if  $R_0 = 1$ , then the fastest  $R_N$  could grow is  $R_0 = 1$ ,  $R_1 = 2$ ,  $R_2 = 3$ ,  $R_3 = 4$ , etc. However, if  $R_0 = 2$ , then  $R_N$  could increase as  $R_0 = 2$ ,  $R_1 = 4$ ,  $R_2 = 6$ ,  $R_3 = 8$ , etc. Based on optimization of  $R$  we desire a geometry that gives  $R_0$  near 2.

## III. SIMULATION

In this section we present the results of numerical simulations for both a single drive bunch and the ramped bunch train of section II passing through the dielectric structure with inner radius  $a$ , outer radius  $b$  and dielectric constant  $\epsilon$  as shown in Fig. 2. Although one would like to design a structure with both high efficiency and rapid acceleration, it turns out that these are competing effects. We therefore begin by examining the trade-off between efficiency and acceleration (i.e. between high transformer ratio and high acceleration gradient.)

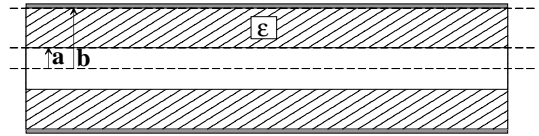


Figure 2) The dielectric wakefield accelerator. A hollow dielectric ( $\epsilon$ ) cylinder of inner radius 'a' and outer radius 'b' covered by a copper covered jacket. The electron bunch passes through the vacuum hole of radius  $a$ .

Rosing and Gai [4] have solved for the longitudinal wakefield in a dielectric structure, excited by an axial current distribution. Using this formalism we present the results of numerical simulations.

We begin by considering a dielectric structure of inner radius  $a = 2.5$  mm,  $b = 2.8$  mm and dielectric constant  $\epsilon = 38$  excited by a single drive bunch with a gaussian axial current distribution. In Fig. 3 we see the longitudinal wakefield excited by the gaussian bunch with  $\sigma = 1.5$  mm.

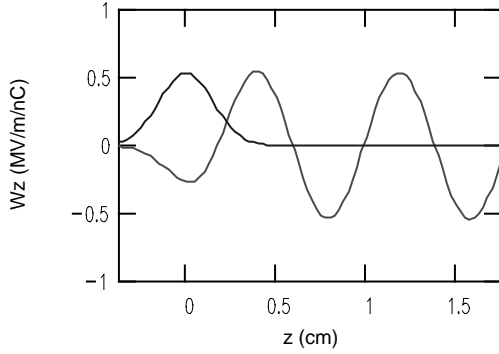


Figure 3) Longitudinal wakefield excited by a gaussian beam with  $Q = 1$  nC and  $\sigma = 1.5$  mm. Note that the fundamental wavelength  $\lambda_0 \approx 8$  mm.

If we define  $E_{\min}^-$  to be the minimum value of the decelerating field within the bunch and  $E_{\max}^+$  to be the maximum value of the accelerating field behind the bunch we define the transformer ratio as  $R = E_{\max}^+ / E_{\min}^-$ . From the calculation used to produce Fig. 3 we see that  $E_{\max}^+ = 0.54$  MeV/m/nC,  $E_{\min}^- = 0.27$  MeV/m/nC, and therefore the transformer ratio  $R_0 = 2$ .

Since  $R_0 = 2$  is not generally true, rather it is a function of the bunch length for a given geometry, we now plot the dependence of  $R_0$  and  $E_{\max}^+$  as a function of the bunch length  $\sigma$  in Fig. 4. It turns out the dependence of  $R$  and the peak gradient  $E_z$  are functions of the ratio of the bunch length  $\sigma$  to the fundamental wavelength  $\lambda_0$  so Fig. 4 is plotted as a function of  $\sigma/\lambda$ .

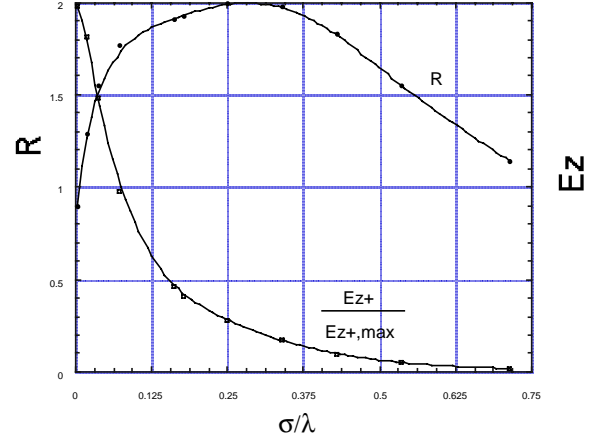


Figure 4) Transformer ratio,  $R$ , and peak accelerating field,  $E_{z+}$  as a function of the normalized bunch length.

From Fig. 4 we see that  $R$  is peaked near bunch length  $\lambda/4$ , but the gradient reaches its maximum when  $\sigma < \lambda/20$ . Therefore, we see there is a tradeoff between efficiency and acceleration gradient. The acceleration gradient is seen to drop off quickly with bunch length, while  $R$  is seen to have a broad maximum. The optimal point of operation is probably near  $\lambda/20$ , but since we are maximizing the transformer ratio, we will choose  $\sigma \approx \lambda/4$ . Now that we have designed a structure single bunch combination to give  $R_0 = 2$ , we proceed with the design of the bunch train according to the rules of section II.

The upgraded AWA facility will be able to produce a 40 nC beam with 1 mm of charge. For our simulations we use a conservative bunch length of  $\sigma = 1.5$  mm. For the dielectric structure under consideration we have  $\sigma/\lambda_0 = 0.2$  which is slightly less than a quarter wavelength. Our desire to operate near a quarter wavelength that drove the choice of  $\lambda_0$  and therefore the dielectric structure. We plot the location of the second bunch relative to the first bunch for  $m = 1$  and therefore  $d = (m+1/2) \lambda_0 = 1.5 \cdot 8 = 12$  mm in Fig. 5. This plot is only to show the position of the second bunch relative to the first bunch - its wakefield is not taken into account in the figure.

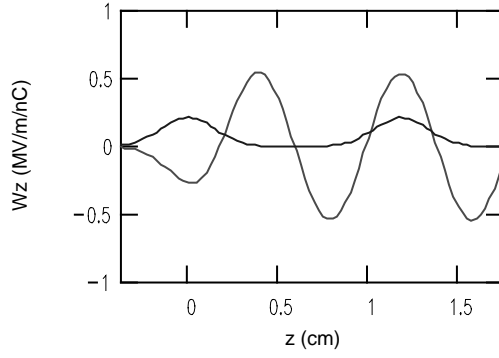


Figure 5) Location of the second bunch in the train for  $m=1$  spacing. The contribution to the net wakefield for the second bunch is not shown.

It is interesting to note that, according to the formulas of Section II, the second bunch is placed in the accelerating phase of the first bunch. The effect of this is to cancel out some of the self-wakefield of the second bunch with the wakefield of the first bunch so that the second bunch experiences a reduced deceleration. The charge of the second bunch is increased until the wakefield it experiences is equal to that of the first bunch. For example, if the first bunch has charge  $Q_0$  and produces fields  $E_{\min,0}^-$  and  $E_{\max,0}^+$  such that  $R_0 = 2$ , then  $E_{\max,0}^+ = 2 * E_{\min,0}^-$ . If the second bunch has the same  $\sigma$  as the first, but a charge of  $Q_1 = k * Q_0$ , then its self-wakefield will be  $E_{\min,1}^- = k * E_{\min,0}^-$ . There-

fore the net wakefield the second bunch experiences  $E = E_{\max,0}^+ - (k * E_{\min,0}^-) = 2 * E_{\min,0}^- - (k * E_{\min,0}^-) = (2-k) * E_{\min,0}^-$ . If this is to be equal to  $E_{\min,0}^-$  then we must have  $(2-k) = -1$  or  $k = 3$ . Finally, we have  $Q_1 = 3 * Q_0$ . This is exactly as specified in Eqn. (2).

The last constraint we must consider for our design is due to the AWA facility. The AWA rf frequency is 1.3 GHz and therefore we can only produce drive beams separated by  $\lambda_{\text{rf}} = 23$  cm. Since our dielectric structure has  $\lambda_0 = 0.8$  cm, then we must operate at  $m^{\text{th}}$  harmonic of  $23/0.8$  or  $m = 28^{\text{th}}$  harmonic.

Using the above parameters, we calculate the wakefield excited by a ramped bunch train (Fig. 1) of charge  $Q_0 = 3$ ,  $Q_1 = 9$ ,  $Q_2 = 15$ , and  $Q_3 = 21$ . The result wakefield is shown in Fig. 6. A close examination of Fig. 6 shows that each bunch experiences the same deceleration wakefield  $E_{\min}^- = -0.8$  MeV/m/nC, while the peak accelerating field behind each bunch  $E_{\max}^+$  increases as  $E_0 = 1.63$ ,  $E_1 = 3.26$ ,  $E_2 = 4.90$ , and  $E_3 = 6.54$  all in MeV/m/nC. This results in a transformer ratio behind each bunch as,  $R_0 = 2$ ,  $R_1 = 4$ ,  $R_2 = 6$ ,  $R_3 = 8$ .

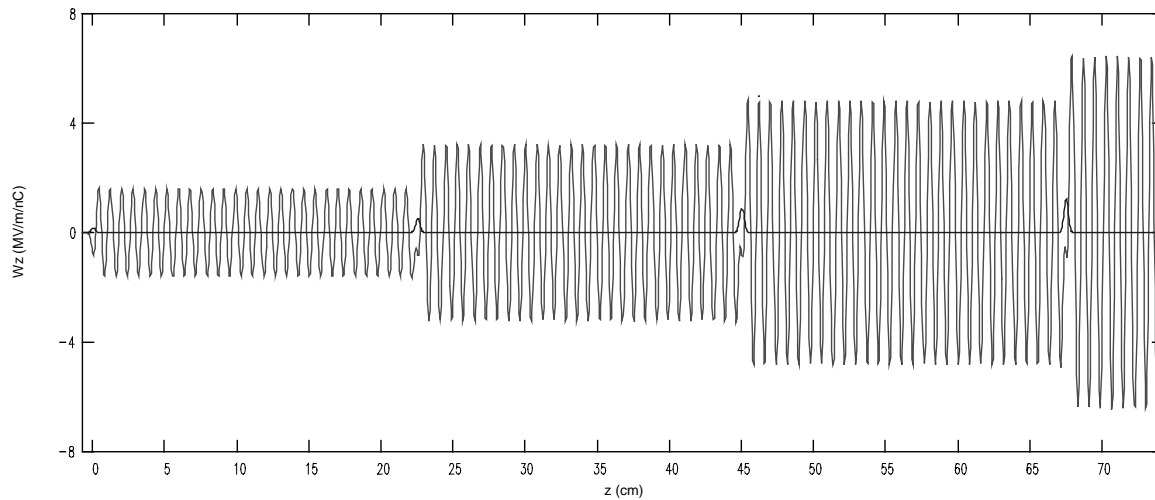


Figure 6) The resultant wakefield produced by a train of 4 electron bunches of bunch length 1.5 mm and charge magnitude  $Q_0 = 3$ ,  $Q_1 = 9$ ,  $Q_2 = 15$ , and  $Q_3 = 21$ . The Transformer Ratio  $R = 8$  for this example.

## IV EXPERIMENTAL SETUP

A train of 4 electron bunches is made by optically splitting a single laser pulse into 4 separate pulses. The distance between bunches is adjusted optically by moving mirrors on translation stages in the delay line. Initially, the distance between bunches is crudely measured with a streak camera, using the 10 ns sweep rate, thus giving us a timing resolution of  $\sim 10$  ps between bunches. Final bunch spacing must be done during the experiment by making the deceleration of trailing bunches equal to the deceleration of the lead bunches after they emerge from the dielectric structure.

To measure the transformer ratio  $R$ , we must infer the deceleration gradient  $E_{\min}^-$  experienced by the four drive beams by measuring their energy loss after emerging from the dielectric structure of Fig. 2. Since all drive beams will experience the same decelerating field, we will only be able to measure the combination of the four beams.

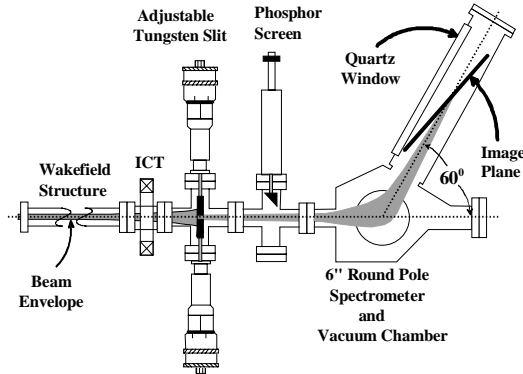


Fig. 7) Energy Measurement System. The decelerated drive bunch train passes through a tungsten slit of adjustable width, which is imaged through a  $60^\circ$  dipole (diameter = 6") onto a phosphor screen located at the image plane. The phosphor screen is viewed with an intensified camera through the quartz window.

Since the group velocity of the rf packet  $\neq 0$ , one must make sure that the length of the tube must be long enough so that the wakefields of the four drive bunches overlap. Since

the dielectric constant is high ( $\epsilon = 38$ ) then we know the group velocity is low;  $\beta_g \approx c/\epsilon = 0.026c$ , where  $c$  is the speed of light. By the time the 4<sup>th</sup> drive bunch enters the tube, the rf packet from the first bunch has traveled a distance  $\beta_g * 3d$  where  $3d$  is the separation between the first and last bunch. Thus  $L$  only need to be greater than  $\beta_g * 3d$  or about 2 cm.

The energy will be measured with the spectrometer shown in Fig. 7. The energy measurement system has a resolution of 0.2% with the tungsten slit set to 300  $\mu\text{m}$ . To complete the measurement of the transformer ratio, one must also know  $E_{\max}^+$  behind each bunch – i.e. one must map out the wakefield left behind the drive bunch train with a witness beam. For a length of tube  $L = \frac{1}{2}$  m, we expect the drive bunches to only lose 0.4 MeV. Thus the drive beam will exit the structure with 15.2 MeV – 0.4 MeV = 14.8 MeV. The witness beam will enter the structure with 4 MeV and receive a maximum acceleration of 3 MeV thus exiting with energy of 7 MeV.

## IV. SUMMARY

We have described an experiment to be preformed at the AWA facility to measure a transformer ratio  $\gg 2$  in a collinear wakefield accelerator. This could have important implications for the future development of any collinear wakefield accelerator.

## V. REFERENCES

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